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Fractal Dimensions as a Model for Qur'anic Semantic Depth: Reconsidering the Concept of Buṭūn of the Qur'an

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ABSTRACT:

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The Qur'an contains verses endowed with profound and complex meanings. Some exegetes and scholars in the field of Qur'anic studies believe that the science of interpretation systematizes this complexity and facilitates deeper understanding. Exegesis, in turn, engages with the various facets or dimensions of the Qur'an. The present paper employs an analytical–comparative method to investigate the issue of *Buṭūn* of the Qur'an (the inward meanings of the Qur'an) and the emergence of new dimensions within Qur'anic verses, while also applying the characteristics of fractal dimensions to the concept of *Baṭn* (inward meaning).

By utilizing the newly articulated dimensions framed within scientific and political *Laffāfeh* (envelope) under the *Buṭūn* of the Qur'an, and by applying the properties of fractal geometry to the Qur'an's inward meanings (*Buṭūn*), the presence of such dimensions can be more readily identified. Thus, it can be argued that the *Buṭūn* of the Qur'an constitute the dimensions of the Qur'an that necessitate its profound conceptual depth. As time progresses and advancements are made, human understanding increasingly reveals that

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the Qur'an, by virtue of these dimensions, has not only remained relevant throughout all eras but has also continuously offered meanings contingent upon humanity's level of understanding, shaped by the achievements and advancements of each age. This profound comprehension stems from God's attribute as the Knower of the Unseen (*ʿālim al-ghayb*).

KEYWORDS: The Qur'an, Multidimensional nature of Qur'anic meaning, Fractal dimensions, *Buṭūn* of the Qur'an, *Laffāfeh* (Envelope).

1. Introduction

The views of exegetes vary across different commentaries; apart from certain verses in which the opinions of some exegetes align, differences in interpretation are generally observable. These divergent perspectives originate from each exegete's specific approach and his particular mode of understanding the verses. Consequently, extensive research has been conducted to analyze the methodological frameworks of these exegetes.

Regarding the conceptualization of *Baṭn*, reference can be made to the article "*The Essence of Baṭn al-Qurʾān and Strategies for Attaining It*" (Rezaei Esfahani 2008). This article discusses the various significations of *Baṭn* and presents Qur'anic, narrative, and rational evidence to validate its existence and the possibility of multiple levels of Qur'anic comprehension. By positing the possibility of non-Infallible individuals attaining the *Buṭūn*, the article outlines specific strategies for accessing inward meanings and regards the *Buṭūn* of the Qur'an as one of the fundamental reasons for the Qur'an's eternal relevance.

The application of the concept of fractal dimensions to the *Buṭūn* of the Qur'an, along with the gradual realization of Qur'anic inward meanings over time, constitutes the primary motivation for the present study. Another objective is to formulate and clarify several existing hypotheses concerning the *Buṭūn* of the Qur'an. Accordingly, this research adopts an analytical–comparative method to propose new hypotheses concerning the Qur'an's inward meanings and to address the following questions by applying the concept of fractal dimensions to the *Buṭūn* of the Qur'an—an approach that has received little prior attention:

How can the concept of fractal dimensions be applied to *Buṭūn* of the Qur'an? What impact does the application of fractal dimensions to the concept of *Baṭn* have on Qur'anic understanding? What relationship exists between existing hypotheses (such as envelope theory), *Buṭūn* of the Qur'an, and fractal dimensions?

2. Fractal Geometry

In modeling many natural elements, Euclidean geometry has proven inadequate, which has paved the way for the emergence of a more advanced and refined geometry known as fractal geometry. Fractal geometry was first introduced by Benoît Mandelbrot in 1975 (Ghobadian 2013, 166). It is utilized to model structures in which similar patterns are repeated across smaller scales (Bovill 1996, 153).

The term fractal originates from the Latin word *fractus* (derived from the verb *frangere*), meaning broken or irregular. Because fractals are structures that exhibit all or some of these characteristic properties, any attempt to provide a single, comprehensive definition inevitably remains insufficient (Pickford 1996, 16). Some of the key features of fractals include self-similarity, pattern repetition, and fractal dimension. The primary focus of this paper is the characteristic of fractal dimensions, which will be examined in detail below.

2.1. Fractal Dimensions

Fractal dimensions lie between integer numbers, meaning that fractals possess non-integer dimensions. For example, a straight line has a dimension of 1, a square has 2, and a cube has 3; however, a fractal can have a dimension between 1 and 2 (Mandelbrot 1983, 147–247).

Understanding the concept of dimension is inherently complex. In the early twentieth century, one of the major challenges in mathematics was defining the notion of dimension and identifying its properties. Since that time, the situation has become even more intricate, as mathematicians have developed multiple concepts of dimension, including Hausdorff dimension, topological dimension, self-similarity dimension, box-counting dimension, and capacity dimension, among others (Peitgen et al. 1993, 202). All these concepts are interrelated; however, some are meaningful only in specific contexts, while others may be entirely irrelevant, thus requiring alternative and more practical definitions. In some cases, these definitions are logically consistent and equivalent, whereas in others, several may be logically valid yet yield conflicting results. As a result, these details can be confusing even for experienced research mathematicians (Peitgen et al. 1993, 202). Accordingly, the present paper does not engage with these technical complexities; instead, it focuses on one of the most widely used methods for calculating fractal dimension: the box-counting dimension.

2.1.1. Box-Counting Dimension

In this method, the fractal shape is covered by boxes of varying sizes. The specific shape of the boxes is not crucial; rather, they must share the same dimensionality as the space in which the fractal is embedded. For instance, fractals on a plane can be covered using solid squares or disks. For spatial fractals, cubic or spherical boxes, or even irregularly shaped enclosures, may be employed (Roozitalab 2010, 48). As the size of the boxes decreases, the number of boxes required to cover the shape increases, resulting in greater detail and more accurate approximations of the fractal structure. Table 1 presents the approximate calculations for the four coverings shown in Figure 1.

Table 1. Approximate box dimensions for a snowflake. As the box diameter decreases, the approximation becomes more accurate (Roozitalab 2010, 48).

Square Diagonal	Number of Boxes N_δ (F)	Square Side Length (δ)	dim_B F ≈ $\frac{\log N_\delta (F)}{-\log \delta}$
0.25	8	0.3535	1.998
0.125	24	0.1768	1.835
0.0625	56	0.0884	1.659
0.03125	136	0.0442	1.575

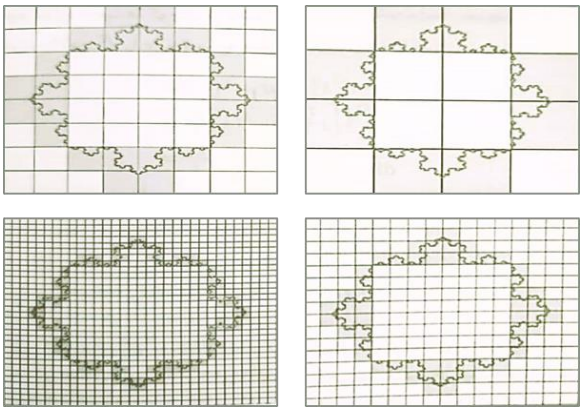

Figure 1. Four different coverings for calculating the dimension of the Koch snowflake using the box-counting method (Roozitalab 2010, 48).

2.1.2. Effective Dimensions

The concept of effective dimension concerns the relationship between mathematical sets and physical objects. In principle, physical entities such as a cloth, a thread, or a small ball should be represented as three-dimensional objects. In practice, however, physicists often treat a cloth, a

thread, or a ball as having effective dimensions of 2, 1, and 0, respectively, provided that their thickness is sufficiently small. For example, in order to describe a thread, theoretical models associated with dimensions ranging from 0 to 2 must be adjusted, revealing minor corrective terms. If conditions are favorable, the model remains operationally valid even when these corrections are ignored. Thus, the effective dimension is fundamentally subjective; it is an approximation and is therefore dependent on the level of resolution (Mandelbrot 1983, 17).

To clarify and substantiate the concept of effective dimension, consider the example of a ball of yarn exhibiting different effective dimensions. A ball with a diameter of 10 cm, composed of thick thread measuring 1 mm in diameter, possesses—by immersion—several distinct effective dimensions.

For an observer situated at a great distance, the ball appears as a zero-dimensional object, that is, a point. (Blaise Pascal and medieval philosophers famously argued that, on a cosmic scale, the entire universe appears as a point.) When observed at a resolution corresponding to a distance of 10 cm, the ball of yarn manifests as a three-dimensional object. At a resolution of 10 mm, it transforms into a tangle of one-dimensional threads. At 0.1 mm, each thread becomes a column, and the entire structure is once again perceived as three-dimensional. At 0.01 mm, each column decomposes into fibers, causing the ball to appear one-dimensional once more. This process continues, with dimensions alternating repeatedly from one value to another. Ultimately, when the ball is represented by a finite number of atom-like points, it returns to a zero-dimensional state.

A similar sequence of dimensional shifts and variations can be observed in a sheet of paper. This principle that numerical results depend on the relationship between the object and the observer, lies at the core of twentieth-century physics and serves as an illustrative example of its foundational assumptions. Most of the objects examined in this paper resemble the ball of yarn in this respect: they exhibit a succession of distinct effective dimensions (Mandelbrot 1983, 17–18).

3. The Concept of Baṭn (Inward Meaning) in the Qur'an

The concept of *Baṭn* (inward meaning) will be examined in both its linguistic and technical senses. The linguistic meaning of *Baṭn* shows little variation across Arabic lexicons and generally denotes what is hidden, in contrast to the apparent (*ẓāhir*) (Ibn Manẓūr 1993, 1: 433; al-Rāghib al-Iṣfahānī 1992, 1: 97; Tabataba'i 2008, 102; al-Zabīdī 1994, 18: 60; al-Ṭūsī 2002, 2: 571; al-Ṭabrisī 1993, 1: 468; Ibn Fāris 1983, 1: 259; al-Farāhīdī

1990, 7: 440; al-Fayyūmī 1993, 52; al-Jawharī 1984, 6: 357; al-Baghdādī 1998, 5: 85; Fakhr al-Rāzī 2000, 8: 172; al-Shawkānī 1994, 1: 375; Hakki 1911, 7: 363; al-Qurṭubī 1995, 4: 178).

From a technical perspective, *Baṭn* has been defined in various ways by researchers and exegetes. Tabataba'i, a contemporary scholar, classified and evaluated these definitions after examining the views of several exegetes and concluded as follows:

It appears that the most comprehensive and accurate theory defines *Baṭn* as "meanings beyond the apparent sense." This view encompasses all implications and necessities of the verses' wording, aligns with the majority of scholarly opinions, and effectively subsumes them. According to this theory, some inner meanings are obtained through reflection and contemplation and, when applied, serve as guidance and instruction for humanity. Other inner meanings are profound and deeply embedded, accessible only through esoteric interpretation (*al-taʾwīl*), and belong to the secrets, subtleties, and truths of the Qur'an. Moreover, this definition is closer to the linguistic meaning of *Baṭn* and stands in contrast to the apparent (*al-ẓahr*) of the Qur'an, which refers to everything manifest, whether in wording or in its immediately perceptible meaning (Tabataba'i 2008, 146).

It is also necessary to note another highly significant technical understanding of *Baṭn* that is frequently cited by numerous exegetes: the identification of *Baṭn* with esoteric interpretation (*al-taʾwīl*). Some exegetes have explicitly equated the concept of *Baṭn* with *al-taʾwīl*. Given that this position is grounded in transmitted narrations, its historical roots are well attested (al-Ālūsī 1994, 1: 8; al-ʿAyyāshī 1960, 1: 11; Qumī 1984, 1: 20; al-Suyūṭī 2000, 2: 6; Maʿrifat 1995, 41; al-Fayḍ al-Kāshānī 1994, 1: 31; Tabataba'i 2008, 103).

Evidence for the existence of *Buṭūn* in the Qur'an includes the Qur'anic verses themselves, narrations, reason (*al-ʿaql*), and the views of scholars and exegetes in this field (Tabataba'i 2008, 151). A broad scholarly consensus, based on these sources, has affirmed—explicitly or implicitly—the existence of *Buṭūn* in the Qur'an (Tabataba'i 2008, 228; al-Khūʾī 1989, 1: 213; al-ʿAyyāshī 1960, 1: 11; al-Fayḍ al-Kāshānī 1994, 1: 31–36; al-Ṭūsī 2002, 1: 9; Tabataba'i 1970, 3: 73; al-Suyūṭī 2000, 2: 265; al-Ālūsī 1994, 1: 8; Tabataba'i Hakim 1993, 1: 95; Ṣadr al-Dīn Shīrāzī 1981, 7: 37–46; Ibn Taymiyyah 1987, 2: 39–40; al-Ghazālī 1937, 1: 289; Maʿrifat 1995, 41).

4. Theory of Qur'anic Laffāfeh (Envelope)

Theory of Qur'anic *Laffāfeh* (envelope), first proposed by Mohammad

Hossein Baroomand, posits that the Qur'an's challenging truths—truths that could not be explicitly articulated at the time of revelation (and for a considerable period thereafter) due to the scientific limitations and political circumstances of those eras—were embedded within multiple layers of meaning to be transmitted to future generations. According to this theory, God placed coverings over certain Qur'anic verses that possess scientific, political, or social dimensions. The purpose of these coverings was to prevent distortion (*al-taḥrīf*) of the Qur'an and to fulfill the divine promise of its preservation (Baroomand 2005, 243). In the case of political and social *Laffāfehs*, it is evident that if a verse provoked the displeasure of ruling authorities, efforts would likely have been made to alter or suppress it. With regard to scientific *Laffāfehs*, some verses contain truths whose understanding and empirical substantiation can only emerge gradually, alongside scientific progress over time. Had these truths been disclosed outside their appropriate historical context, they would have been incomprehensible and might have exposed the Qur'an to criticism or rejection, owing to their unverified nature at the time. Thus, these envelopes can be understood as mechanisms safeguarding the Qur'an from textual corruption.

With continued and deeper engagement with the Qur'an, new forms of envelopes may be identified. This observation indicates that the diversity of envelopes expands in proportion to the multiplicity of Qur'anic examples (Joudavi & Faqihi 2024, 365). According to Baroomand's theory, each envelope comprises three principal components:

1. The Covering: a layer that conceals the meaning, such as a shell, veil, or outward form.
2. The Core: the concealed element beneath the covering, such as its essence or kernel.
3. The Contextual Clue: the indicator(s) required to penetrate the covering and access the concealed meaning.

Baroomand argues that God employed these envelopes mainly in two domains: first, political matters, especially verses concerning the Prophet's family (*Ahl al-Bayt*), and second, scientific subjects that exceeded the intellectual horizon of the original audience. Explicitly stating such scientific truths could have caused misunderstanding, led people to reject the Qur'an, or allowed opponents to exploit the verses for propaganda (Joudavi & Talebian 2024).

The role of contextual clues in emerging from the envelope is fundamental. Wherever God employs an envelope, He simultaneously

provides clues for its comprehension, which can be discovered through contemplation and reflection. Truth-seekers are thus able to identify these clues within the verses and gradually peel back the envelopes.

The primary reason God employs envelopes is the divine promise of preserving the Qur'an: *"Indeed We have sent down the Reminder, and indeed We will preserve it"* (Q. 15: 9). This divine promise operates within, not outside, the causal system governing the universe, and its fulfillment does not depend on coercion or supra-natural intervention. Rather, the Wise God realizes this promise through natural means. Just as Moses (PBUH) was preserved through ordinary causes, God generally avoids disrupting the natural causal order, except in exceptional cases to complete the proof against specific individuals.

One of the natural mechanisms for preserving the Qur'an is eliminating the motivation for literal corruption and preventing unauthorized manipulation. By refraining from explicitly stating certain truths, and by employing concealment over others, God reduced the incentive for direct textual alteration of the Qur'an. These truths were embedded within envelopes so that only those who engage deeply and contemplatively with the divine verses could attain God's intended meanings and the depths of the Qur'anic message. By presenting superficial truths and apparent meanings, God directed the attention of the general audience toward these accessible aspects. In contrast, the core truths and divine intent require penetrative insight and transcending initial appearances.

In the search for envelopes within the Qur'an, it must be emphasized that no subject should be imposed or justified arbitrarily. For example, in relation to the verses Q. 5:3 and Q. 33:33, many exegetes have faced interpretive difficulties, with some proposing that the placement of these verses may be incorrect in order to resolve perceived inconsistencies. Accepting such a claim for even a single verse would call into question the order and preservation of the entire Qur'an. However, the hypothesis of political envelopes provides a coherent explanation for this issue. Whenever interpretation encounters difficulty, one must assume the presence of a concealed point. Through reflection, the subtle and elegant truths of the Qur'an are gradually unveiled.

Regarding the means of accessing envelopes, these may be divided into apparent and inward methods. The apparent means include contemplation and reflection upon the verses, as well as the identification of contextual clues; moreover, narrations frequently provide primary guidance for uncovering concealed meanings. The inward means consist of purification and spiritual self-refinement. It may be assumed that bi-dimensional verses

simply contain two equally accessible meanings. However, the distinction lies in the fact that, in such verses, God does not intend for one meaning to conceal another. Instead, two independent meanings are conveyed through brevity, enabling the maximum conveyance of meaning with minimal wording.

There is also a conceptual relationship between envelopes and the doctrine that the Qur'an possesses multiple inward meanings (*Dhū Buṭūn*). The term *Dhū Buṭūn* refers to the existence of successive layers of meaning, comparable to the layers of an onion, where each layer reveals another beneath it. By contrast, when one meaning conceals or masks another, the phenomenon is described as an envelope (Joudavi & Talebian 2024). Nevertheless, all these multi-meaning phenomena—bi-dimensionality, multi-facetedness, multiple *Buṭūn*, and envelopes—share a common feature: they all demonstrate the multidimensional nature of Qur'anic meaning. Accordingly, this study examines several scientific and political envelopes as illustrative cases.

4.1. Example of a Scientific Envelope

Numerous examples of scientific miracles in the Qur'an have been proposed by various scholars (Barati 2022; Moradi 2025). The Earth's motion constitutes another such example, which simultaneously illustrates the concept of envelopes. For thousands of years, humanity believed the Earth to be stationary, with the Sun, Moon, stars, and planets revolving around it, as articulated in the Ptolemaic model. In the sixteenth century, Copernicus proposed a heliocentric system in which the Sun occupies the center and the Earth, along with other planets, revolves around it. Galileo's telescopic observations, including the moons of Jupiter and the phases of Venus, provided empirical support for this theory.

The Earth moves continuously, yet no sensation of motion or vibration is perceived, much like passengers aboard a ship traveling at constant speed. Despite this reality, when Galileo articulated this view, efforts were made to silence him, as the historical context was unsuitable and public comprehension insufficient. Remarkably, the Qur'an articulated this truth much earlier, though its message was preserved within an envelope until it reached its appropriate historical audience:

"And you see the mountains, which you suppose to be stationary, while they drift like passing clouds the handiwork of Allah who has made everything faultless. He is indeed well aware of what you do" (Q. 27: 88).

In its primary and apparent meaning, analogous to the first fractal

dimension, this verse appears within the context of passages addressing the Day of Resurrection and thus evokes the condition of mountains at that time. However, closer examination reveals that the movement described is gradual, unlike other eschatological verses such as: *"when the mountains are set moving"* (Q. 81: 3). This constitutes a second level of meaning, comparable to a second fractal dimension.

The verse thus conveys a scientific reality that was beyond the epistemic horizon of its initial audience. Consequently, its apparent meaning functioned as an approximation, analogous to a dimension between 1 and 2 in fractal theory. Through this envelope, the verse remained preserved until its intended time of disclosure, when contextual clues and scientific advancement unveiled its deeper meaning. This unveiling resulted in scientific faith—the recognition that the articulation of such knowledge fourteen centuries earlier was humanly impossible, thereby affirming the Qur'an's miraculous nature.

At first glance, the verse may now appear to clearly indicate the motion of the Earth. However, a widely accepted principle in Qur'anic studies is that verses are embedded within a context (*al-siyāq*), meaning that a cluster of verses typically addresses a single thematic subject. More precisely, verse Q. 27: 88 is subtly positioned among verses related to the Resurrection, and the reference to the Earth's motion is introduced with remarkable delicacy from multiple angles (Baroomand 2005, 243). This structural placement led the audience of the time to assume that the described motion would occur during the Resurrection. Nevertheless, a subsequent clue is embedded within the verse to indicate that this motion is unrelated to the Resurrection: *"…the handiwork of Allah who has made everything faultless"* (Q. 27: 88).

This clause at the end of this verse functions as a contextual indicator, enabling the reader to exit the dominant eschatological context. Upon careful reflection, it becomes evident that God's handiwork (*ṣunʿ Allāh*) and ongoing creation are not confined to the Day of Resurrection; on the contrary, during the Resurrection, annihilation prevails and creative processes cease. Moreover, the depiction of mountains that appear rigid yet are simultaneously in motion is incompatible with Resurrection imagery, wherein mountains are described as disintegrated, scattered, and flowing: *"and the mountains are shattered into bits, and become scattered dust"* (Q. 56: 5-6) (Joudavi & Faqihi 2024, 366).

Baroomand offers another illustration of a scientific envelope, stating: It is noteworthy that, unlike scientific pioneers and geniuses who often generate upheaval and controversy in order to assert their discoveries, the bearer of the Qur'an, in this and similar instances, deliberately avoids

commotion. Out of necessity, a suitable covering is placed over the truth so that the unprepared minds of contemporaries are not disturbed. When conditions mature, scientific advancement unveils the concealed truth, thereby demonstrating the veracity of the Qur'an to scientists and others alike. Importantly, an appropriate clue is always embedded to facilitate the recognition of the envelope in such cases. At times, the erroneous scientific assumptions held by people have rendered the use of envelopes unnecessary, as in the case of the Sun's longitudinal motion mentioned in verse Q. 36:38, which for an extended period was interpreted according to the incorrect astronomical models prevalent at the time (Baroomand 2005, 243).

4.2. Example of a Political and Social Envelope

The Verse Q. 5:3, known as the verse of completion (*al-Ikmāl*), contains the rulings on certain forbidden foods and also expresses the perfection of religion:

> *You are prohibited carrion, blood, the flesh of swine, and what has been offered to other than Allah, and the animal strangled or beaten to death, and that which dies by falling or is gored to death, and that which is mangled by a beast of prey barring that which you may purify and what is sacrificed on stone altars [to idols], and that you should divide by raffling with arrows. All that is transgression. Today the faithless have despaired of your religion. So do not fear them, but fear Me. Today I have perfected your religion for you, and I have completed My blessing upon you, and I have approved Islam as your religion. But should anyone be compelled by hunger, without inclining to sin, then Allah is indeed all-forgiving, all-merciful (Q. 5:3).*

Beyond these apparent injunctions on prohibitions, however, Shi'i sources, citing numerous Shi'i and Sunni reports, link the occasion of revelation of the verse to the event of *Ghadīr Khumm*, interpreting "perfection of the religion" as referring to the proclamation of Imam ʿAlī's (PBUH) succession and the establishment of divinely guided leadership (*al-wilāyah*) (Amīnī 1995, 1:447–456). Nevertheless, multiple explanations have been proposed concerning the placement of the phrase of perfection of the religion among such legal rulings, a matter that remains one of the principal points of disagreement among exegetes.

According to Baroomand's theory of Qur'anic envelopes, this placement is a deliberate political envelope designed to protect the Qur'an from distortion. Opponents of ʿAlī would naturally read "completion of religion" as referring to the legal injunctions (the first fractal dimension). However, given the occasion of revelation of the verse and the contextual clues, it becomes clear that the true completion of religion lies in the succession of

ʿAlī (corresponding to the second fractal dimension). In fact, the apparent meaning conveys an approximate sense that is neither fully the first meaning nor fully the second (a non-integer dimension between 1 and 2).

Here the decisive contextual clue is the phrase "*Today the faithless have despaired of your religion. So do not fear them, but fear Me*" (Q. 5:3) which appears immediately after the legal injunctions. This phrase enables readers to move beyond the immediate legal frame: the legal rulings are subsumed under the broader concept of rulership (*al-wilāyah*), itself a form of divine ordinance. It is implausible that religion would be completed merely by ordinary legal rulings, many of which appear elsewhere in the Qur'an, so the completion must concern a more foundational matter, namely the appointment of a successor. Thus, God embedded the ruling on appointing a successor within seemingly secondary legal material, and the clause about disbelievers' despair functions as the contextual clue signaling that the intended meaning extends beyond legal injunctions.

5. Application of Fractal Dimensions to Buṭūn of the Qur'an

Just as mathematicians, over time and through scientific advancement, have identified multiple conceptual dimensions within fractals, exegetes have likewise continued to uncover ever-deeper inward meanings (*Buṭūn*) of the Qur'an. To expand the scope of this analogy, one may consider the correspondence between fractal dimensions and the hypothesis of Qur'anic envelopes, which itself may be regarded as one of the *Buṭūn* of the Qur'an.

Accordingly, the Qur'an—with its vast semantic, epistemological, and philosophical layers—can be likened to a high-dimensional fractal, such that the closer one approaches it, the more previously unseen dimensions are revealed. This correspondence finds support in the box-counting method (Figure 1), which demonstrates that as the number of boxes increases, the box diameter decreases, yielding greater detail and a more accurate approximation of the underlying structure. This mechanism parallels the envelope theory, wherein contextual clues guide the interpreter toward the core semantic layer of a verse. Put simply, the presence of a clue within an envelope enables a more precise approximation of the verse's intended meaning.

The discussion of effective dimensions, though originating in purely scientific discourse, aligns well with the diversity of perspectives held by both the general public and exegetes regarding the Qur'an. To clarify this point, one may consider the contrast between a layperson's perspective and

that of a specialized researcher with respect to the Qur'an, or indeed, any complex subject. The layperson resembles a distant observer, perceiving a ball of yarn as a zero-dimensional point, whereas the researcher, depending on their level of expertise, observes the same object from a significantly closer and more informative distance.

5.1. Modeling the Qur'an's Semantic Depth Based on Fractal Dimensions

The primary objective of this study is the conceptual modeling of the phenomenon of *Buṭūn* of the Qur'an through the characteristic of fractal dimensions. Just as mathematicians in fractal geometry have achieved multiple conceptual and computational dimensions (such as Hausdorff dimension and Box-Counting dimension), with the passage of time and scientific advancements, exegetes discover more inward meanings of the Qur'an. This modeling rests on two fundamental axes: the variation of effective dimension, and the increase in detail, understood as a progressively more accurate approximation of meaning.

5.2. Buṭūn of the Qur'an as Infinite Fractal Dimensions

From this perspective, the Qur'an may be understood as a high-dimensional fractal, encompassing immense semantic and philosophical depth. A defining feature of fractals is their non-integer dimensionality, which lies between whole numbers and represents levels of complexity that are neither purely linear (dimension 1) nor purely planar (dimension 2). In a similar fashion, the *Baṭn* of a verse is neither limited to the apparent meaning (low dimension) nor entirely detached from it; rather, it constitutes an intermediate semantic layer characterized by non-linear complexity.

As one approaches this fractal structure, namely, the Qur'an, and increases the scale of interpretive engagement, new dimensions emerge that were previously imperceptible. This ongoing process of discovery offers a compelling conceptual explanation for the infinite nature of *Buṭūn*, as explicitly referenced in transmitted narrations.

5.3. Explaining Buṭūn through the Concept of Effective Dimension and the Observer

The notion of effective dimension, as introduced by Mandelbrot, provides a powerful analytical framework for understanding the *Buṭūn* of

the Qur'an. Effective dimension is not fixed, but rather depends on the relationship between the object and the observer:

- Distant Observer (Superficial Understanding): An ordinary reader or an exegete operating in an earlier historical period with limited epistemic tools resembles an observer viewing a ball of yarn from afar. At this scale, the object appears as a zero-dimensional point, and the verse yields only its surface or apparent meaning.
- Close Observer (Deep Understanding): A modern researcher or exegete, equipped with advanced scientific, linguistic, and philosophical tools, analogous to altering scale and magnification, is able to discern the inner structure of the ball of yarn, such as the interwoven fibers of the thread, and thus perceive higher dimensions. Similarly, scientific and intellectual progress alters the interpretive scale, enabling the effective dimension of the verse, its *Baṭn*, to increase for the exegete.

This gradual shift in effective dimension provides a rational foundation for the envelopes hypothesis. The envelopes, manifested through contextual clues, function as interpretive magnifying instruments that, in every era, allow the observer to derive increasingly precise approximations and richer details of the verse's core semantic content.

5.4. Applying Contextual Clues to the Box-Counting Method

The applicability of fractal dimensions to the envelope hypothesis is further substantiated through the box-counting method. In this method (Figure 1), reducing the size of the boxes, that is, refining the measurement scale, results in an increased number of boxes, which in turn yields greater detail and more accurate approximations of the fractal structure. Analogously, within the framework of Qur'anic envelopes, scientific, linguistic, and textual clues function as these smaller boxes. Each clue effectively refines the scale of interpretation, enabling the exegete to arrive at increasingly precise approximations of the *Buṭūn* concepts, namely, the core semantic layer of the verse. This dynamic explains how a fixed and immutable text, the Qur'an, can continuously disclose new semantic dimensions, as human knowledge progresses and the interpretive scale of the observer contracts. Thus, the box-counting model provides a rigorous conceptual justification for the infinite semantic productivity of the Qur'an without implying textual alteration or instability.

6. Conclusion

Based on the preceding discussion concerning *Buṭūn* of the Qur'an and the application of fractal-dimension characteristics to the Qur'an's inward meanings, the following conclusions may be drawn. The concept of fractal dimensions, which possesses diverse and extensive conceptual facets, is meaningfully applicable to *Buṭūn* of the Qur'an, due to the structural alignment and internal coherence among multiple semantic layers of Qur'anic meaning. Applying fractal dimensions to *Buṭūn* of the Qur'an provides a time-sensitive interpretive framework, reminding interpreters that human comprehension operates at varying levels of scale. This framework allows for progressively more accurate approximations of meaning, in accordance with epistemic, scientific, and methodological advancements across historical periods. The hypothesis of the Qur'anic envelopes (*Laffāfeh*), which is classified under the concept of *Buṭūn* of the Qur'an, exhibits a direct correlation with varying levels of human comprehension across different eras. The envelopes neither negate the apparent meaning nor replace it, but instead regulate access to deeper layers of meaning through contextual clues. *Buṭūn* of the Qur'an encompass numerous instances, including newly identified examples discussed in this study. By modeling *Buṭūn* of the Qur'an through a fractal system, this research offers a novel theoretical framework demonstrating that the Qur'an's inward meanings function as a coherent, multi-dimensional semantic structure.

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